SYMMETRY ANALYSIS AND GEOMETRIC MODELLING

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ABSTRACT

This paper discusses a new method of detecting symmetry in two- and three-dimensional objects. A multi-dimensional approach is used to interrogate a set-theoretic representation of an object. A method for detecting planes of true symmetry is described, as well as more advanced methods for partial symmetry, rotational symmetry and partial rotational symmetry. The examples given in the paper are two-dimensional but the methods for three-dimensional evaluation are discussed.

1. INTRODUCTION

Finding symmetry in engineering components and other objects is of great importance for a wide range of applications. This paper takes an engineering point of view of symmetry, but many of the benefits of the method it proposes carry across to other disciplines. Iwanowski[1] says that the evaluation of symmetry is an NP-hard problem, and that efficient algorithms exist, but are limited in their use. This has not discouraged a great deal of subsequent research being carried out in the field.

Symmetry analysis can be applied to a wide range of object descriptions. For example, Kuehnle[2] applied a symmetry-based recognition scheme to locate the backs of road vehicles in images. Marola[3][4] and, more recently, Pei[5] have also concentrated on the symmetry in images. Others, such as Atallah[6] and Davis[7], have used evaluation techniques for images composed of line segments, circles and points. However, by far the largest area of research is in evaluating the symmetry in point sets. Highnam[8], Wolter[9] and, more recently, Lin[10] have developed a number of techniques which have shown good results for simple objects. All of the methods mentioned above deal with either true symmetry or rotational symmetry about an object's centroid; and some allow limited partial symmetry analysis of simple objects.

2. TYPES OF SYMMETRY

There are a number of possible symmetries that can be defined (Figure 1):

- True symmetry, where the object is divided in two through its centroid, each side being the mirror image of the other.

- Partial symmetry, where the object is not truly symmetrical, but planes exist near its centroid that are very close to producing mirror images.

- Rotational symmetry, where portions of the object can be seen to repeat at angular intervals (for instance a gearwheel).

- Partial rotational symmetry, where portions of the object can be seen *approximately* to repeat at angular intervals.

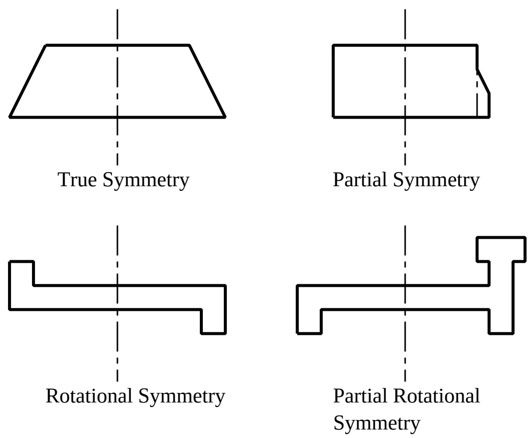

Figure 1: Types of Symmetry.

3. SYMMETRY HIERARCHY

Symmetry can be seen as a hierarchical description of an object. An object may have a number of primary symmetries: planes that split the object in half through its centroid, and may also have secondary symmetries: planes that split the object in half but require analysis of the resultant portions (Figure 2). Finding hierachical symmetry requires a model of the object to be recursively analysed until a non-symmetric portion is reached. By storing the various symmetry planes found on the way, along with the final non-symmetric portion, the total amount of information required to represent the object can be greatly reduced. In the case of perfect symmetry the original model can be reinstated from this information as and when required.

4. SYMMETRY DETECTION

The symmetry-detection process that we propose is based on the premise that for a symmetry to take place a second image of the model must fit on to the original model. We measure the exactness of such a “fit” is the length of perimeter of the combined model and second image at the fit position. If the perimeter length of the combined model is equal to that of the original model then a perfect match has been located. If the perimeter length is less than the original model then a best-fit percentage can be calculated. The object description that we use is a set-theoretic solid model. This type of description can easily represent one-, two-

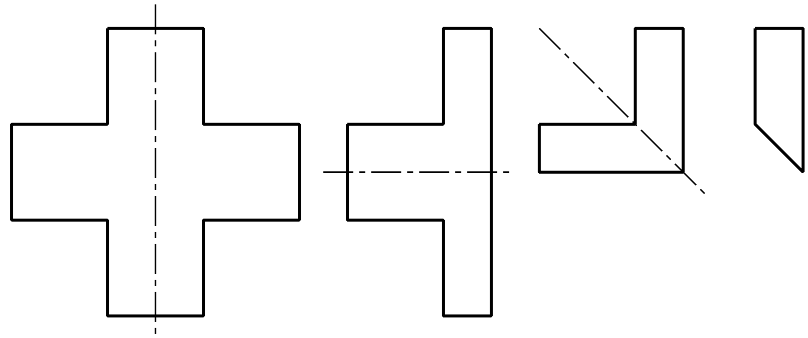

Figure 2: Symmetry Hierarchy.

or three-dimensional objects.

5. MULTIDIMENSIONS

Woodwark's multi-dimensional approach[11] has been programmed to manipulate set-theoretic models. The multi-dimensional description of the model (which we call a hypermodel), allows it to exist in all possible positions and rotations in an *object space*[1]. To make this clear, consider the case of a one dimensional set-theoretic model, which can be represented by an interval on a line ([3,4] on Figure 3). If we wish to fit a *template* to this model, the template needs to be able to translate across the object space (here the object space is the real line from 0 to 6). The resultant hypermodel in this simple case is two-dimensional: one original dimension, and one hyperdimension, representing how far the template has been translated. The translation appears as a diagonal sweep in a second dimension (Y). The Y coordinate measures the displacement of the template. The model which does not move is represented by a vertical sweep in the second dimension. Now consider the intersection of the model and pattern (the dark-grey region). For a match to take place, two corners of the intersection area must exist at the same height in the second dimension, as shown by the line Ym. The positions of these corners are located by using a recursive binary division of the hypermodel. The method is an enhancement of an existing technique for feature recognition also developed by the authors. A more detailed description of this is given in the authors' ealier paper[11].

For the case of true symmetry of a two-dimensional model one hyperdimension is required: that of rotational angle about the model's centroid in the $X - Y$ plane. For partial symmetry detection the partial symmetry planes may not pass through the model's centroid, so two further hyperdimensions are required. These two extra hyperdimensions are translation along the x axis and translation along the y axis. This allows the pattern to "float" across the object space, which the algorithm searches for a fit position.

[1] generally a hyperbox completely enclosing the hypermodel.

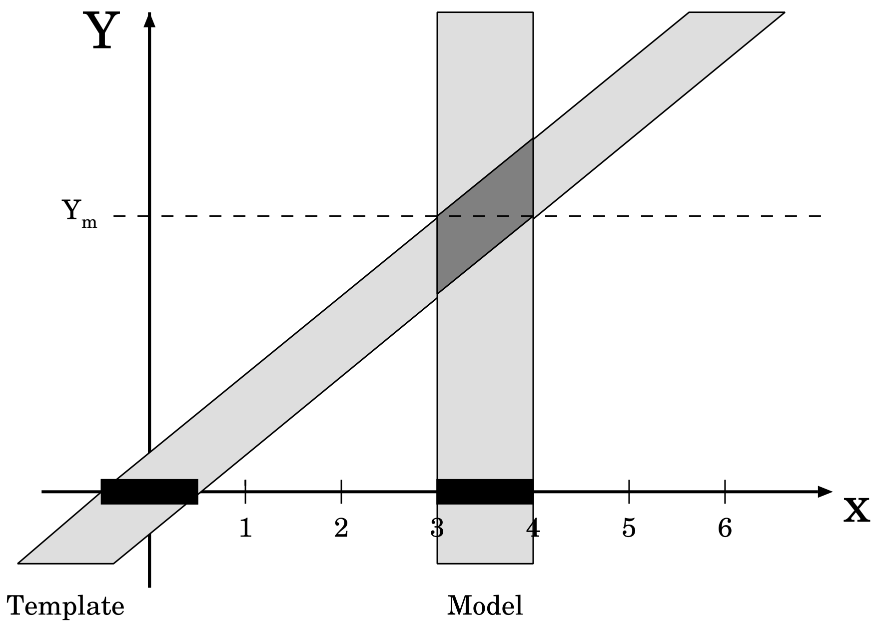

Figure 3: Hyperdimensions.

True symmetry and partial symmetry require a template that is a mirror image of the original component. For convience the component is mirrored about its centroid using the X axis as the mirror plane. For both of the rotational symmetries, the template is an exact copy of the component; there is no need for any type of transformation.

All of the object solid models used for this paper are two-dimensional for ease of display, but the process does extend to allow symmetry detection for three-dimensional objects and we are in the process of invsetigating this.

6. RESULTS

A number of two-dimensional components were modelled to test the system. Some of the models were strictly symmetrical, and some had partial symmetry planes. The analysis initially calculated the surface area, perimeter length and centroid of the component. It was then translated so that its centroid coincided with the origin of the object space. For true symmetry detection the mirror image of the component was swept through the rotational hyperdimension to encompass rotations between 0 and π. Figure 4 shows the results of a test on a component[1] that is truly symmetrical. The original component is shown along with its hierarchical decomposition.

To detect partial symmetry the method had to utilise the translational as well as the rotational hyperdimensions. Again the component was translated to place its centroid at the origin, but the mirror image was allowed to float across the

[1] Part of a tape-cassette transport mechanism

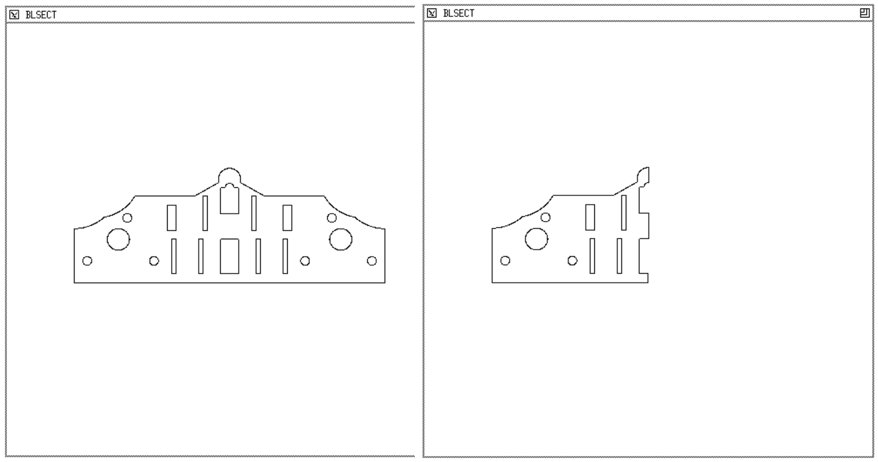

Figure 4: True Symmetry.

whole object space as well as rotating between 0 and 2π. The method displays the original unsymmetrical model along with the two symmetrical models that could be constructed by mirroring the two halves of the model through the partial symmetry plane (Figure 5). The detection of partial symmetries allows designers to reconsider the structure of the complete model. A decision can then be made as to which of the three models is the best for its required function. For example. A designer may decide to make an object with partial symmetry completely symmetrical, as that may make it easier to manufacture. Alternatively, the designer may wish to exaggerate slight asymmetries, so as to make the object easier to orient uniquely.

Detecting rotational symmetry differs from the method described above as it entails a straight copy of the component to be used for the template, rather than a mirror image. Figure 6 shows the results obtained for rotational symmetry, and Figure 7 shows the results of the partial rotational techniques.

7. CONCLUSIONS

The methods developed can detect both the hierarchical symmetry and partial symmetry of two-dimensional set-theoretic models. The components to be evaluated can be constructed with a few simple straight edges or from many highly complicated polynomials; the only disadvantage of the latter is increased computation time. The accuracy of the evaluation can be easily modified by altering the size of the smallest division boxes in the recursive binary division of the hypermodel. Reducing the division box size has the effect of increasing the accuracy, but reduces the speed of computation.

8. FURTHER WORK

At present the methods for partial symmetry detection are time-consuming, due to the object division being carried out in five dimensions. As it is known that the partial symmetry planes will pass through or near the component's centroid,

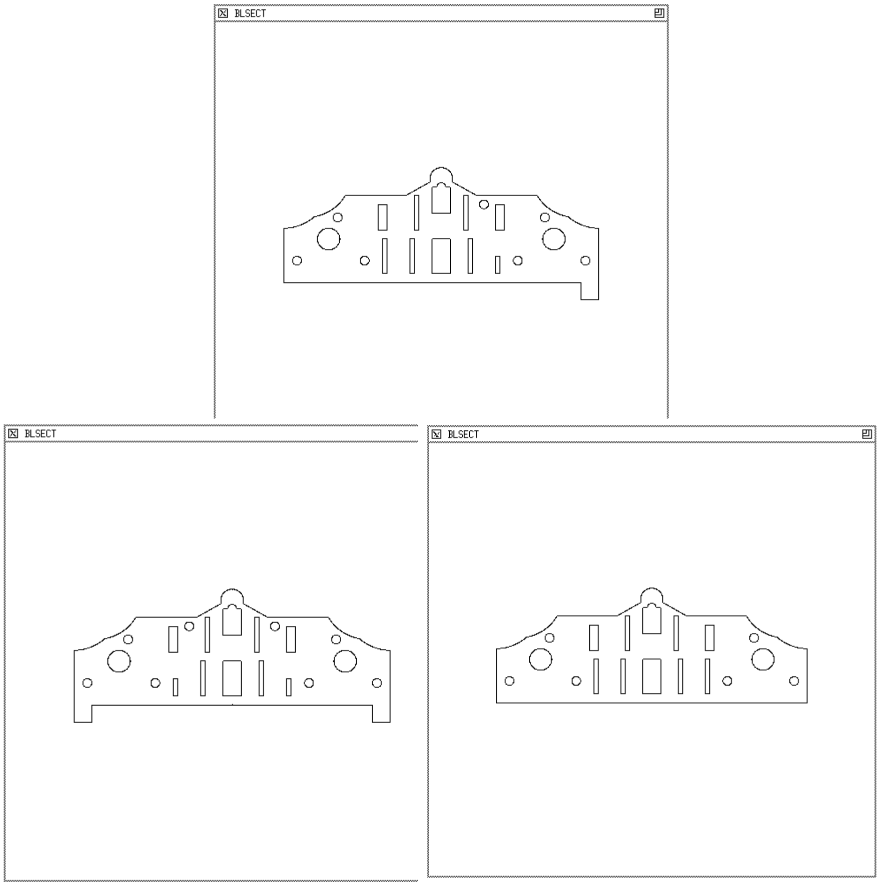

Figure 5: Partial Symmetry.

the search space of the higher dimensions can be reduced to investigate only the area in the immediate vicinity of the centroid rather than the whole object space.

A working system for symmetry detection in three-dimensional components is currently being constructed; this will entail two more rotations and one more translation, and hence a nine-dimensional hypermodel.

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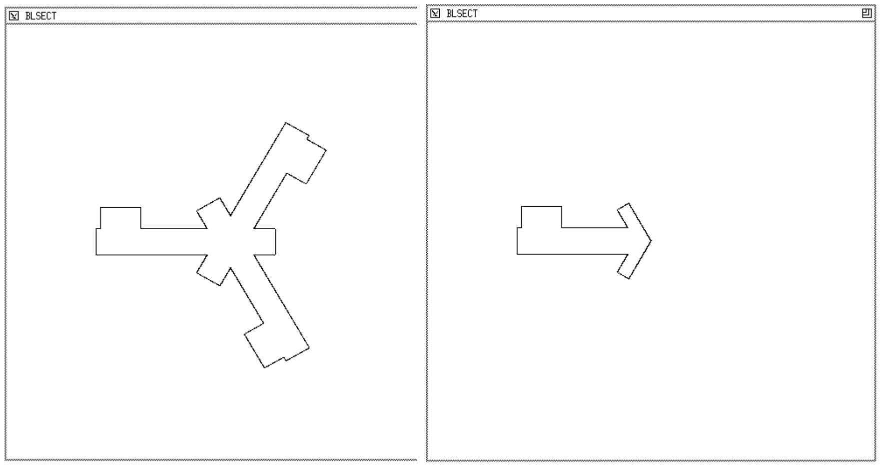

Figure 6: Rotational Symmetry.

10. REFERENCES

[1] Iwanowski S.,"Testing approximate symmetry in the plane is NP-hard", Theoretical Computer Science. Vol.80, 1991, pp.227-262.

[2] Kuehnle A.,"Symmetry-based recognition of vehicle rears", Pattern Recognition Letters, Vol.12, April 1991, pp.249-258.

[3] Marola G.,"Using Symmetry for Detecting and Locating Objects in a Picture", Computer Vision, Graphics, and Image Processing, Vol.46, 1989, pp.179-195.

[4] Marola G.,"On the Detection of the Axes of Symmetry of Symmetric Planar Images", IEEE Transactions on Pattern Analysis and Machine Intelligence, Vol.11, No.1, January 1989, pp.104-108.

[5] Pei S. and Lin C.,"Normalization of Rotationally Symmetric Shapes for Pattern Recognition", Pattern Recognition, Vol.25, No.9, 1992, pp.913-920.

[6] Atallah M.J,"On Symmetry Detection", IEEE Transactions on Computers, Vol C-34 No. 7, July 1985, pp. 663-666.

[7] Davis L.S.,"Understanding Shape: II. Symmetry", IEEE Transactions on Systems, Man, and Cybernetics, Vol.SMC-7, No.3, March 1977, pp.204-212.

[8] Highnam P.T.,"Optimal Algorithms for Finding the Symmetries of a Planar Point Set", Information Processing Letters, Vol.22, No.4, April 1986, pp.219-222.

[9] Wolter J.D., Woo T.C. and Volz A.,"Optimal Algorithms for Symmetry Detection in Two and Three Dimensions", The Visual Computer, Vol.1, 1985, pp.37-48

[10] Lin J., Chou S., and Tsai W.,"Detection of Rotationally Symmetric Shape Orientations by Fold-Invariant Shape-Specific Points", Pattern Recognition, Vol.25, No.5, 1992, pp.473-482.

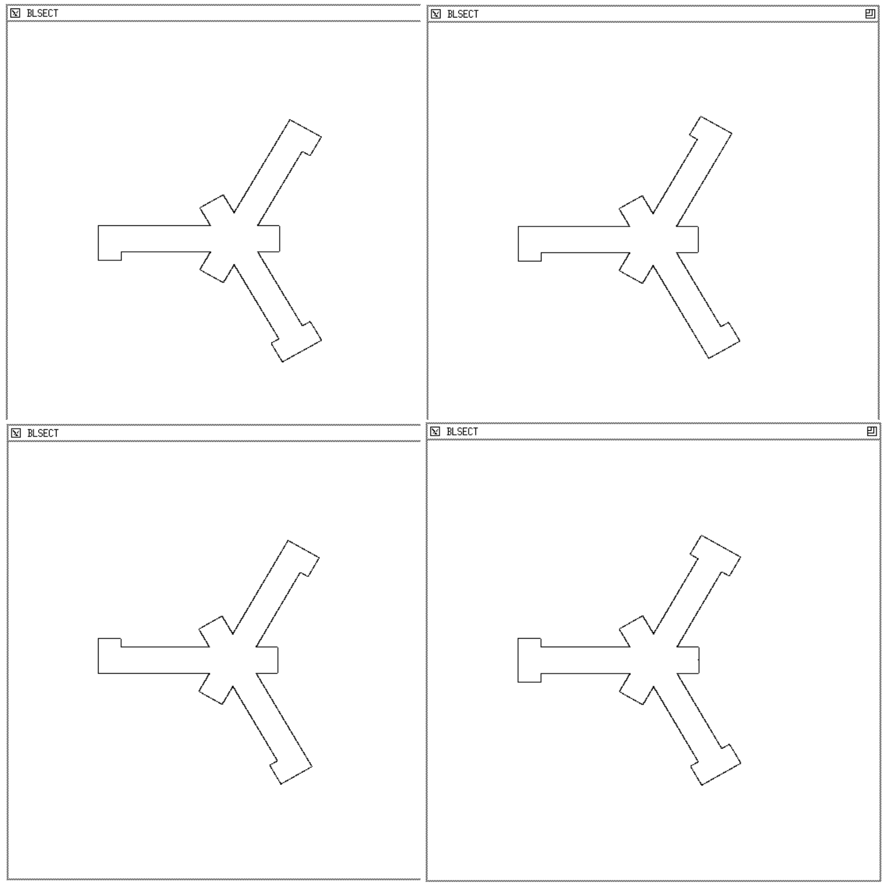

Figure 7: Partial Rotational Symmetry.

[11] Parry-Barwick S. and Bowyer A.,"Woodwark's Method for Feature Recognition", Bath University Technical Report 099/1992. Submitted to Computer-Aided Design 1993.